A Consideration of the Functional Roles of the Valuation Parameters: the Linkage of the Flow and the Stock Models

Aiichiro Suzuki

Abstract

The objective of the paper is having an insight for the OJ Model as of its structure with regard to the following questions in comparison to the residual earnings model by Ohlson[1995]:

(1) the reason for not having "other" information

(2) only two periods' expecting profit without setting up the terminal value will suffice for the valuation

Searching the reasons for those questions might also provide the clues for another question about the treatment of an interest rate movement for the valuation process without falling back on the stochastic process.

I　Introduction：　The Objective of the Paper

The Objective of the paper is to find out the clues for the questions with regard to the structure of the Ohlson Juettener Nauroth Model, or the OJ model[1] in short, through examining the framework of another famous valuation model by Ohlson(1995).

Generally speaking, in dealing with the valuation for the continuous payoffs such as dividends or cash flows, you might not be able to avoid facing the problem in determining the discount rate which is specifically called the issue of estimating the cost of equity capital.

However, the cost of capital is strongly related to each firm's strategy in a sense in distributing a limited amount of the company resources that involves so much uncertainty and that are beyond an accounting information in the corporate valuation models.

Consequently, more often than not, even though the discounting factor plays an role of lynch pin, it oftentimes be treated like a "given" or "fixed" number. Is it safe enough for implementing an accurate valuation?　That's the starting point of the discussion.

Going back to the subject of the OJ Model as of its structure, it may well to notice the following two questions will pop up in comparison to the Ohlson's original residual earnings model：

(1) the reason for not having "other" information

(2) only two periods' expecting profit without setting up the terminal value will suffice for the valuation

Considering the reasons for those questions might also provide the clues for the question about an interest rate movement. Therefore, what this paper focuses will not only on an interest rate movement but some issues to be extended relating to those two questions above.

Ohlson started to make the discussion based on the two fundamental forms of the valuation

---

[1] See Ohlson et.al [2006]

models based on the properties of an accounting information specifically called stock and flow.

"The stock model" is the one based on the cumulative amount of the discounted residual earnings with the book value along with the clean surplus assumption. The Ohlson model is one of these kinds. Meanwhile, "the flow model" uses only the information of the earnings. The OJ Model, is one of the examples.

Ohlson contemplates the theoretical environment in accordance with the roles of accounting information through those two basic fundamental models called the M-M and the E-S models.

First, in the Mark to Market Model or the MM model in short, a corporate value, commonly may be represented by its stock price, is always equal to the book value in this environment. In other words, all information to affect the valuation will directly and instantaneously reflected on the book value.

In the meantime, the Earnings Sufficiency Model or called the E-S model, only the discount value of the cumulative residual earnings will be engaged in the valuation process. The Ohlson model, in short, can be enunciated as the combination of the M-M and the E-S model.

Ⅱ The M-M Model

Taking a precise look at each model through the following equations will be conducive to a better understanding. Firstly, the M-M model is shown in the equation (1).

$$E_t[\tilde{x}_{t+1}] = r_t b_t = r_t P_t \qquad (1)$$

In the equation, "t+1" means the expecting profit that is unknown from the point of "t". However, the book value “b” is equivalent to the market price “P”. Then, any changes in an accounting information will be reflected on the book value immediately. If there is any time lag, the variance that will be supposed to account for the amount of good-will. However, in this case, no presence of good-will will be perceived.

$$E_t[\tilde{x}^a_{t+1}] = 0 \qquad (2)$$

In the valuation process, the residual income will be realized exactly as previously expected. And that creates the expecting value of the residual income to be zero. It is shown in the equation (2).

Overall, in the M-M model, only the stock information not the flow information will be involved in the valuation process.

Ⅲ The E-S Model and the Formation of the Multiplier

Next, we will see the E-S model that uses only the information of the earnings. The point is its earnings multiplier functioning to capitalize the subsequent earnings into the corporate value.

In this model, the Modigliani Miller theorem (called the MM theorem) plays an important role. It sets force the condition the dividends has nothing to do with the outcome of the valuation. In equation (3), "f (·)" will be the multiplier.

$$P_t = f(.)x_t - d_t = f(.)[x_t + r_{t-1}z] - [d_t + (1+r_{t-1})z] \quad (3)$$

$$f(.)r_{t-1}z - (1+r_{t-1})z = 0 \quad (4)$$

$$f(.) = (1+r_{t-1})/r_{t-1} \quad (5)$$

The equation (4) shows the dividends will be reserved and will be re-invested out of that reserve. Additionally, the situation, due to the MM theorem, the amount of the reserve and of re-invested has no variance. But the point is not the amount but the sub-indices to be attached to z. Making a comparison of those double zs shown in the parenthesis, the variance is zero and that situation creates the multiplier to be shown in the equation (5).

Ⅳ The Multiplier for Forecasting the Value

$$P_t = (R_{t-1}/r_{t-1})x_t - d_t \quad (6)$$

$$P_t = E_t[\tilde{x}_{t+1}]/r_t \quad (7)$$

In the equation (6), the sub-indices attached to "x" shows the time is the present (t) whereas that of the multiplier in the parenthesis is that of the past (t-1).

In the equation (7), the time period for the numerator is stochastic[2]. But the denominator is a fixed number not stochastic even if it relates to the future event. The clue for considering the reason could be found in the multiplier in the equation (6). The key of the idea is a time lag.

In the previous case, it was the past to the present, however, in this case it is a current period to the next. When there is a time-lag, you don't have to worry about it to be stochastic or not.

Ⅴ The E-S Model and the Inheritance of the Past Information

In terms of the inheritance of the past information to the present, the comparison between the M-M and the E-S will be shown in the equation (8) which shows the theoretical structure of the M-M model.

$$x_t = r_{t-1} P_{t-1} + \tilde{\varepsilon}_t \quad (8)$$

$$x_t = r_{t-1} P_{t-1} + (r_{t-1}/R_{t-1}) \tilde{\varepsilon}_t \quad (9)$$

---

[2] The status of being stochastic is indicated by tilde(~).

P is the price that is equivalent to the book value since anything happen, a direct transition in no time is the rule and that made it have no time lag between P and the multiplier (r). However, there should be the variance of the amount between the planned and the realized. That will never be known until the present.

Because of that situation, the parameter of ε (epsilon) is attached to absorb the difference as an error term which can be called the "white" noise meaning no trend or tendency is observed in terms of the pattern of the occurrence of such errors. Without any multiplier to the parameter of ε (epsilon), the information about the error term in the previous period will be perfectly inherited.

In the equation (9), again, the variance between the planned and the realized is also attached. But the multiplier attached is the reverse number. It is based on the suppositional condition that we are already in the future to look "back" into the present. Accordingly, the time that multiplier indicates should be backward. That situation indicates t-1 is involved. But it is only through the analysis of the related equations. In reality, however, the sub-indices of "t" not just "t-1" will be involved.

Obviously, it has a contradiction caused by the fact the analysis is made only by the earnings not by the residual earnings. This is called the theorem of reduction of absurdity meaning to prove something not true indirectly by presenting something to cause a contradiction. In this case the M-M model is not true. However, the purpose of doing so is to compare the multiplier with that of the E-S model through the inheritance of the past information as of the MM model, it shows the perfect inheritance of the past information which is unlikely to happen. Meantime, the multiplier in the E-S model shows only a partial inheritance will take place due to the time lag.

It triggers to consider how the multiplier has come into being; the difference of the numerator and the denominator is caused by the time lag, not by the difference of the amount. The point is without a time lag, the multiplier will eventually lose its meaning. The implication of the observation is, as the limitation of a numerical analysis, the sub-indices for ε (epsilon) shows only t-1, however, ε (epsilon) was derived from the comparison of the valuation of the previous and the present, so the sub-indices of "t" must be needed. And that could be one of the reasons to use the residual income rather than using only the income.

In due course, we will see the residual income next instead of the income.

Ⅵ The E-S Model and the Residual Earnings

Naturally, the application of the residual earnings to the E-S model will be considered at first.

$$\tilde{x}^a_{t+1} = x^a_t + \tilde{\varepsilon}_{t+1} \qquad (10)$$

$$E_t[\tilde{\varepsilon}_{t+1}] = 0 \qquad (11)$$

$$\tilde{x}^a_{t+1} = \omega_t x^a_t + \tilde{\varepsilon}_{t+1} \qquad (12)$$

The equation (10) shows the situation the cost of equity never changes. Additionally, in the equation (11), the attaching error term ε (epsilon) has the sub-indices indicating the time is "t+1" meaning the actual number is unknown because it relates to a future event.

To support the condition, the parameter is stochastic but the expecting mean value is zero as the expectation should always coincide with the outcome. The interpretation of the equation (10) is without having any multiplier, it turns out to be stochastic and it follows a random walk process. Without doubt, we should avoid facing the event and that makes us attach the parameter whose value must not be 1 that is shown in the equation (12).

The point is again, first, to deny the possibility for being a random walk process. Second, having a time lag shown in the parameter that was created by taking advantage of the nature of the residual earnings which is to compare the present with the future. By doing so, the parameter of ω which was originally meant to be the duration of the residual earnings, but it turns out to be the multiplier for the valuation model.

Reviewing the points above, the E-S model is the model to make the earnings into the valuation. It must have something to capitalize the information of the earnings into the outcome. Also the multiplier to be attached ought not to be zero.

Next, we will consider the reason for another key question: the reason for the flow model to work without having “other” information unlike the stock model like the Ohlson Model.

Ⅶ A consideration for “Other” Information

Again, we will see the ordinary income not the residual to be seen in the equation (13).

$$P_t = k\left(\frac{R}{r}x_t - d_t\right) + (1-k)b_t + \beta\,\upsilon_t \qquad (13)$$

$$\tilde{x}^{a}_{t+1} = \omega\, x^{a}_{t} + \upsilon_t + \tilde{\varepsilon}_{1,t+1} \qquad (14)$$

$$\tilde{\upsilon}_{t+1} = \gamma\,\upsilon_t + \tilde{\varepsilon}_{2,t+1} \qquad (15)$$

The reason for doing so is probably we don’t need to consider the complicated relations of each component such as the treatment of the dividend to be re-invested or not. In this equation, the three parts which is the profit, the book value and "other" information, will simply be laid out. But it has a deeper insight more than just simplicity: to correspond “k” which used to be nothing but a distributing ratio of the M-M and the E-S models with “ω”. It shows originally the duration of the residual earnings, that makes it possible to think about the structure of the model without being bothered by the relation of those components in the way how each of them will give an impact for its valuation.

We also need to mention the necessity for inspecting the validity of its auto regression process by examining of the multiplier to be stochastic or not. That means it will not be enough to look only the value of the multiplier; the time to be shown in the sub-indices needs to be stressed. Without a time lag, the E-S model never works. As previously mentioned, the changes of an interest rate from "t-1" to "t" was explained by attaching the parameter in the

form of a fraction whose numerator is either a period ahead or behind to the denominator. Anyhow, that time lag makes the parameter work. Otherwise, the equation will fall into a random walk and that makes the model virtually impossible to make a further analysis.

In the equations (14) and (15), the information dynamics showing how "other" information will be attached to the Ohlson Model. Those also give us an additional implication about the roles of the parameters introduced in the equations. As reviewed all those points above, in those equations also show the same situation by attaching the sub-indices of "t+1" not just "t" to the future residual income since it is unknown at the present. And the error term or to be called the “white” noise is also attached.

By the way, what does "other" mean? It is something other than an accounting information. The parameters of ω and γ, both of them are originally meant to be the duration of the numbers to be attached. Those parameters, at the same time, will be attached to the equations in the form of an auto regression process. They both assume the function to deny the equation to follow the random walk process unless whose value are happened to be 1.

$$P_t = k\ (\frac{R_{t-1}}{r_{t-1}} x_t - d_t) + (1-k\ )b_t + \beta \upsilon_t \qquad (16)$$

$$\omega_t = \frac{r_t}{r_{t-1}} \frac{r_t + 1}{r_t + k} k \qquad (17)$$

$$\gamma_t = R_t - \frac{r_t + k}{r_t \beta} \qquad (18)$$

The equation (16) is made out of the earnings rather than the residual earnings but this time, the sub-indices of the time period is attached. Another objective by showing in the equation (16) is to make a further analysis of those parameters of ω and γ.

Specifically, both of the parameters will be expressed by the other parameters that are k and β. That are shown in the equations (17) and (18). The parameter of k originally means the distribution ratio of the M-M and the E-S models whereas β is the durability of "other" information.

However, in this case, staying away from the original property, those will be used for the sensitivity test to see the influence of an interest rate change on the information content of the residual earnings in terms of its timing in a sense how near or far from now.

Ⅷ The sensitivity Analysis Using the Parameters of ω and γ

$$\partial \omega_t / \partial r_t > 0 \qquad (19)$$

$$\partial \omega_t / \partial r_{t-1} < 0 \qquad (20)$$

$$\partial \omega_t / \partial k\ > 0 \qquad (21)$$

The equations (19) and (20) show the result of the sensitivity test for only to the parameter of ω not for β.[3]

The analysis shows ω will be larger than zero under the condition of the present rate whereas it will be less than zero with the previous rate. It means as for the sensitivities of the parameter of ω experiencing the change of an interest rate, it will be more sensitive if it is at the present than in the past. Those are shown in the equations (19) and (20).

In the equation (21), ω increases its amount in line with the changing of the parameter of k meaning the information contents of the income gets larger as the timing of an interest rate gets closer to now, rather than back into the past.

We also have to mention another important implication; the information contents of the residual earnings which will be represented in ω will also gets larger as the information contents of the earnings grows which is shown in k, now we can confirm that the residual earnings and the earnings are closely related. This is important because ω is related to the behavior of the residual earnings and k is to the earnings and that ω and k can now be used inter-changeably. That is the bottom line.

Concerning the equation (21), as the timing get closer to the present from the past, the information contents of the earnings grows. That gives the foundation for creating the multiplier consisting of the numerator and the denominator whose timing is by a period lagged each other. The parameter ω representing the persistence of the residual earnings has even more strong effect than that of the earnings which is shown in the equation (21). The point is, again, the parameter ω can be substituted for the parameter of k.

IX ω&θ Show High Sensitivities toward an Interest Rate Movement

$$E_t[\tilde{x}_{t+1}] = \omega(x_t + r\Delta b_t) + (1-\omega) r b_t \qquad (22)$$

$$E_t[\tilde{x}_{t+1}] = \theta_t(x_t + r_t\Delta b_t + \%\Delta\ r_t\ \ x_t) + (1-\theta_t) r_t b_t + \upsilon_t \qquad (23)$$

$$\theta_t = \frac{1+r_t}{k+r_t} k \qquad (24)$$

In the equation (22), the expecting income will be explained by the income and the book value using the parameter ω as the distribution ratio. In the equation (23)、in stead of tacking the time sub-indices on to ω, another parameter θ is introduced. After introducing θ, the equation will be accompanied with "other" information resulting to express the Ohlson Model.

The point in this expression is θ which has the function to show the distribution ratio of which the value is obviously not 1. That prevents the equation from following the random

[3] As the flow model does not have “other” information, we do not need anything about β.

walk process either.

Another point is for both of the equations (21) and (22), the first term of the right hand side of those equations have the information of the book value. Those are the parts not to be absorbed into the book value since those equations are made out of the E-S model concept. It shows a clear-cut distribution of the amount of the M-M and the E-S models. However, it is hardly be able to make it even if the parameter of the distribution ratio has been introduced.

Still, that is not to be a crucial issue. The point is, instead, because of the expecting income, during the lapse of the time from "t-1" to "t", the amount of θ grows in line with the increase of an interest rate. In other words, as time goes by, the information contents of the E-S model will be opulent.

The implication of the mechanism is under the condition of a high interest rate, the accounting information contents will decrease which is one of the reasons why the model needs to have "other" information.

However, in the flow model which lacks the book value as well as "other" information, the valuation process must be implemented only through an accounting information.

Probably, one of the answers to beat such a strict restriction is deemed to be the OJ model.

X The Additional Triple Weighted Average Model

$$P_t = w_{1,t} b_t + w_{2,t}\left(\frac{R_{t-1}}{r_{t-1}} x_t - d_t\right) + w_{3,t}\frac{E_t[\tilde{x}_{t+1}]}{r_t} \qquad (25)$$

$$w_{1,t} = (1 - k - \beta r_t + \beta \omega_t r_{t-1}) = (1-k)\left(1 - \beta r_t \frac{r_t}{r_t + k}\right) \qquad (26)$$

$$w_{2,t} = (k - \beta \omega_t r_{t-1}) = k\left(1 - \beta r_t \frac{r_t + 1}{r_t + k}\right) \qquad (27)$$

$$w_{3,t} = \beta r_t \qquad (28)$$

Now we have to get rid of "other" information from the OJ model. Ohlson presented another suppositional equation which is the triple weighted average of the three elements that are the book value, the earnings and the expecting income shown in the equation (25).

The purpose of the analysis is to consider the linkage of the interest movement and that of the expecting profit. Skipping the proof process, let's take a look at the result directly.

The higher the interest rate, the lower the amount of the term with the distribution parameter w1, w2. However, only the one with w3 will increase to be shown in the equations through (26) to (28).

In short, increasing an interest rate will make the information contents of accounting scanty whereas that of the expecting profit will become rich.

The result will be extended to apply the OJ model which obviously spares "other" information that is supposed to fill the gap of something an accounting information misses.

Again, that makes it possible to use the expecting profit without being bothered by paying attention to "other" information.

Although this is nothing but an analogical reasoning lacking an empirical back-up, it ought to be the foundation for the clue to the question presented at the beginning not to have "other" information in the flow models.

The problem about dealing with the movement of an interest rate, the answer can be found in another question : why we don't have to worry about the stochastic process of an interest rate changes, contrariwise, pay all attention to just two consecutive periods movement? The answer is, again, to be shown in the parameter of ω.

Ⅹ Ⅰ The Versatility of the Parameter ω

$$P_t = \frac{R_{t-1}}{r_{t-1}} x_t - d_t \quad (29)$$

In the equation (29), there is the multiplier whose timing will be a period lagged behind to that of the earnings. The timing of the interest rate to be applied by the number is that of the previous term. It leads to a sort of the self explanatory conclusion: only two periods of information can make sense to let the parameter work and that suffices for the valuation.

$$\omega_t = \frac{r_t}{r_{t-1}} \quad (30)$$

The equation (30) shows ω itself. The denominator of "R" is comprised of 1 and r, in due course, "r" which is the numerator will function to realize the value. The point of the mechanism of the parameter of ω is taking advantage of the time lag in comparison of the expecting and the realized to create the variance which will be construed as the residual income. And that settles to require the information for only two consecutive terms.

In the equation (30), ω shows the duration of the residual income. The parameter of tilde (~) with the error term indicates the equation is about the future event whose outcome will never be known until now.

By the same token, the expecting residual income has also the parameter of tilde. The equation holds the form of an auto regression process but by attaching the parameter of ω whose amount must not be 1 in order for the equation to stay out of being stochastic. It is consistent with the nature of the residual income described above. In other words, only two consecutive periods' information will be enough for the valuation expurgating a random walk process from the formation of the residual earnings.

Ⅹ Ⅱ The Conclusive Remark

Wrapping up the entire discussion, the first point is the parameter of ω that will play the two different roles other than the duration of the residual income:

(1) the denial of random walk process of the equation to be attached
(2) makes it possible for the information for only two consecutive terms to suffice for the valuation which makes the OJ model come into being without considering the stochastic process.

Next point is the expecting profit that should be the precursor of the valuation to hold what beyond an accounting information stands for in the model and that makes it possible without having "other" information in the flow models.

The two questions with regard to the point how the OJ model has been formulated have now found their clues through examining the back ground of the Ohlson's residual earnings model.

Reference

Helfert,E.A. [2000] Techniques of Financial Analysis, 10th ed. McGraw-Hill.

Ohlson, J.A. and Z. Gao [2006] Earnings, Earnings Growth and Value, Now Publishers.

Fama, E.F [1981]"Stock Returns, real activity, inflation and money," American Economic Review, 71 (4), pp.545-565.

Gebhardt W.R.,C.M.C.Lee and Swaminathan B. [2001] "Toward an Implied Cost of Capital," Journal of Accounting Research, Vol. 39, No. 1, pp. 135-176

Ohlson,J.A.[1989]"Accounting Earnings, Book Value, and Dividends: The Theory of Clean Surplus Equation (Part1)," Working paper, Columbia University. Reprinted in Brief P, Peasnell K (1996) Clean surplus: a link between accounting and finance. New York and London, Garland Publishing.

----------[1995]"Earning, Book Values, and Dividends in Equity Valuation," Contemporary Accounting Research, Vol. 11, No. 2, 1995 , PP. 661-687.

----------[1998]"Cash Flow Analysis and Equity Valuation,"AIMR Publication.

----------and B. Juettner-Nauroth [2005] "Expected EPS and EPS Growth as Determinant of Value ," Review of Accounting Studies, 10(2-3), pp. 349-365.

----------and D.Gode [2002]"Accounting-Based Valuation with Changing Interest Rates," Review of Accounting Studies, Vol. 9, Number 4, pp.419-441.

----------and D.Gode [2000]"Valuation, Linear Information Dynamic, and Stochastic Discount Rates," New York University Working Paper No. 2451/27481

Penman S.H. and Nissim D. [2003] "The Association between Changes in Interest Rates, Earnings, and Equity Values," Contemporary Accounting Research, Winter 2003, Volume: 20 Issue: 4 pp.775-804

Fukui Y. [2008] Kaikei Sokutei no Saihyoka, Chuo Keizai Sya

Suzuki A. [2009] "Kaikei Joho to Kigyo Kachi Hyoka Model – OJ model no kouzou to Oyo Kanousei "Kigyo Kaikei Vol. 61.3 pp142-148

Suzuki A.［2010］ “Seicyo Model Kara Kieta Sonota Joho” Seijyo Daigaku Kiyo, Vol10、pp59-70
Fujii H., Yamamoto T. [1999] “Kakei Joho to Cash Flow Joho no Kabuka Setumeiryoku ni kansuru hikaku kennkyu－Ohlson model tekiyo to kaizen no kokoromi” Kaikei Vol.116.2, pp170-185
Noma M.,Inoue T［2007］ “Takakuka Senryaku to Kabushiki Shihonn Kosuto” Shoken Analyst Journal Vol.45.10 pp84-97

[謝辞]
本稿は、星城大学経営学部研究費の成果の一部である。
また、本稿は、日本会計研究学会(JAA)の助成により、台湾会計研究学会(TAA) の年次大会（2012年 12 月、国立台湾大学）で行った報告をベースとした。記して謝意を表する。